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## OPEN BRST ALGEBRAS, GHOST UNIFICATION AND STRING FIELD THEORY

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Geometrical aspects of the BRST quantization of charged antisymmetric tensor fields and string fields are studied within the framework of the Batalin and Vilkovisky method. In both cases, candidate anomalies which obey the Wess-Zumino consistency conditions are given.

### 1. Introduction

The quantization of actions invariant under gauge symmetries with a BRST algebra which closes only up to classical equations of motion [1] has recently attracted some interest [2–5]. In order to associate to such symmetries a nilpotent BRST operator, Batalin and Vilkovisky have developed a lagrangian formalism which builds on earlier developments in a hamiltonian framework [6]. They associate for each one of the classical, ghost and antighost fields, collectively denoted by  $\phi$ , an anti-field  $\phi^*$ . This doubling permits the construction of a nilpotent BRST operator,  $s$ , which acts on all fields,  $\phi$  and  $\phi^*$ , together with an  $s$ -invariant local action  $S(\phi, \phi^*(\phi))$ . The anti-fields are not quantum fields. They are to be eliminated through the choice of a local gauge function  $\psi^{-1}(\phi)$  by using in the action and the transformation rules the constraint  $\phi^* = \delta\psi^{-1}/\delta\phi$ . After elimination of the anti-fields one finds an action,  $S(\phi, \phi^*(\phi))$ , which is gauge fixed and BRST invariant, and contains, in general, higher-ghost interactions. The elimination of the antifields leads to new BRST transformations which are not necessarily nilpotent. However, Batalin and Vilkovisky have shown formally that only the unphysical quantities depend on the choice of  $\psi^{-1}(\phi)$ , and that the quantum theory generated by  $S(\phi, \phi^*(\phi))$  leads to a unitary and gauge invariant  $S$ -matrix provided the Ward identities which follow from the new BRST invariance can be enforced order by

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order in perturbation theory [1]. In general, the action  $S(\phi, \phi^*(\phi))$  cannot be obtained by a Faddeev-Popov type procedure.

The construction of a nilpotent BRST operator by doubling the degrees of freedom is reminiscent of a trick which has been widely used in mathematics, in particular by Alain Connes in the framework of noncommutative geometry. Consider for instance a non-nilpotent square matrix  $Q$ . Clearly the  $2 \times 2$  block matrix

$$\begin{pmatrix} Q & i\sqrt{Q^2} \\ i\sqrt{Q^2} & Q \end{pmatrix}$$

is nilpotent. Thus by a doubling of the representation space, a non-nilpotent operator has been transformed into a nilpotent one.

The aim of this paper is to show that, despite the doubling of the degrees of freedom in the Batalin-Vilkovisky formalism, there exists a geometric structure which is of the same type as that encountered in the ordinary BRST formalism for the gauge theories of forms [7]. By geometric structure we mean the formulation of the BRST transformations as curvature constraints in an enlarged space such that the classical and ghost fields are unified, a feature which is allowed by defining a grading which is the sum of the ghost number and the ordinary form degree [7]. A well known motivation for such a geometrical formulation is that it simplifies the classification of anomalies through descent equations [7, 8]. In this paper, we shall show that the geometric formulation does indeed exist in two examples: the Freedman-Townsend model [9] which is the simplest gauge theory of a charged form with degree larger than one, and string field theory [10–12]. In particular, in the former example we will find candidate anomalies, and in the latter example we will be led to a reinterpretation of string fields with negative ghost number, which have been previously interpreted as antighosts, as anti-fields and to the introduction of new degrees of freedom which are the true antighost string fields.

## 2. The charged antisymmetric tensor

We consider the case of a 2-form gauge field  $B_2 = B_{\mu\nu} \frac{1}{2} dx^\mu \wedge dx^\nu$  valued in the Lie algebra  $\mathcal{G}$  of a given Lie group, and possibly coupled to a 1-form field  $A = A_\mu dx^\mu$  also valued in  $\mathcal{G}$ . The spacetime dimension  $d$  is chosen to be 4. The following classical action has been introduced by Freedman and Townsend [9]

$$I_{\text{cl}} = \int d^4x \operatorname{Tr} \left( \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_{\rho\sigma} + \frac{1}{2} A_\mu A^\mu \right), \quad (2.1)$$

where  $F = dA + A \wedge A$  is the field strength of  $A$  and the trace is over the adjoint

representation of the Lie group. The field equations are

$$F_{\mu\nu} = 0, \quad \epsilon^{\mu\nu\rho\sigma} D_\nu B_{\rho\sigma} + A^\mu = 0. \quad (2.2)$$

By solving the field equation  $F = 0$  one obtains  $A = g^{-1} dg$ . By substitution into (2.1) one finds the classical equivalence between the Freedman-Townsend model and a sigma model on a group manifold.  $I_{\text{cl}}$  is invariant under the following gauge transformations

$$\delta B_{\mu\nu} = D_{[\mu} \epsilon_{\nu]} \equiv \partial_{[\mu} \epsilon_{\nu]} + [A_\mu, \epsilon_\nu], \quad \delta A_\mu = 0, \quad (2.3)$$

where  $\epsilon_\mu$  is a local  $\mathcal{G}$ -valued infinitesimal 1-form parameter. Notice that the field strength  $G = DB_2$  is gauge covariant modulo the field equation of  $B$ :

$$\delta G = [F, \epsilon_\mu dx^\mu]. \quad (2.4)$$

Although the structure of the gauge transformation (2.3) seems to be an Abelian one, the gauge algebra is degenerate since  $\epsilon_\mu = D_\mu \epsilon$ , for any  $\mathcal{G}$ -valued scalar parameter  $\epsilon$ , is a zero mode of the gauge transformations modulo the field equation  $F = 0$ .

Before the work of Batalin and Vilkovisky the quantization of  $I_{\text{cl}}$  has been studied [7, 13, 14], the conclusion being that the following spectrum of  $\mathcal{G}$ -valued fields is necessary

$$\begin{array}{ccc} B_0^0 & & \Pi_0^1 \\ B_1^{-1} & B_0^{-2} & , \quad \Pi_1^0 \quad \Pi_0^{-1}, \quad A. \\ B_2^0 & B_1^1 & B_0^2 \end{array} \quad (2.5)$$

In this notation, the upper label shows the ghost number  $g$  and the lower label indicates the degree  $l$  of the form. The sum of both labels is the total degree of the object. The exterior derivative  $d = dx^\mu \partial / \partial x^\mu$  and the BRST operator  $s$  have  $g = 0$ ,  $l = 1$  and  $g = 1$ ,  $l = 0$ , respectively. Therefore both of them are odd operators. Furthermore they are assumed to anticommute,  $(sd + ds) = 0$ . The baseline of the large triangle in (2.5) contains the main fields: the original classical field, its ghost, and the ghost for ghost. We call this sector the *geometric sector*. The other fields are not geometrical in the sense that they belong to the antighost sector, and are in one to one correspondence with the Stueckelberg type fields  $\Pi$  displayed in (2.5). The latter ones will be used as Lagrange multipliers to impose gauge conditions on the 2-form gauge field  $B_2$  and its 1-form ghost  $B_1^1$  and antighost  $B_1^{-1}$ .

The on-shell “BRST operator”,  $s_0$ , which is nilpotent modulo the field equation  $F = 0$ , is defined by

$$\begin{aligned} s_0 B_2^0 &= -DB_1^1, & s_0 A &= 0, \\ s_0 B_1^1 &= -DB_0^2, \\ s_0 B_0^2 &= 0; \end{aligned} \tag{2.6a}$$

$$\begin{aligned} s_0 B_1^{-1} &= \Pi_1^0, & s_0 \Pi_1^0 &= 0, \\ s_0 B_0^{-2} &= \Pi_0^{-1}, & s_0 \Pi_0^0 &= 0, \\ s_0 B_0^0 &= \Pi_0^1, & s_0 \Pi_0^1 &= 0. \end{aligned} \tag{2.6b}$$

One can verify that  $s_0^2 = 0$  on all fields except  $B_2^0$  for which  $s_0^2 B_2^0 = -[F, B_2^0]$ . For the geometric sector it is meaningful to define the following unified object

$$\mathcal{B} \equiv B_2^0 + B_1^1 + B_0^2. \tag{2.7}$$

Note that the total degree,  $l + g$ , of each field on the right-hand side is equal to 2. It is also meaningful to unify the operators  $s_0$  and  $d$  into  $s_0 + d$  [7]. One can thus define the generalized curvatures

$$\mathcal{G} \equiv (d + s_0)\mathcal{B} + [A, B] = \mathcal{D}\mathcal{B}, \tag{2.8}$$

$$\mathcal{F} \equiv (d + s_0)A + \tfrac{1}{2}[A, A]. \tag{2.9}$$

Using the assumption that  $ds_0 + s_0 d = 0$ , one has  $(s_0 + d)^2 = s_0^2$ , and the Bianchi identities for  $\mathcal{F}$  and  $\mathcal{G}$  read

$$\begin{aligned} (d + s_0)\mathcal{G} &= s_0^2 \mathcal{B} + [\mathcal{F}, \mathcal{B}], \\ (d + s_0)\mathcal{F} &= s_0^2 A + [\mathcal{F}, A]. \end{aligned} \tag{2.10}$$

One can easily verify that the geometrical BRST transformations (2.6a) are equivalent to the following constraints on the generalized curvatures

$$\begin{aligned} \mathcal{G} = G &\equiv dB_2 + [A, B_2], \\ \mathcal{F} = F &\equiv dA + \tfrac{1}{2}[A, A], \end{aligned} \tag{2.11}$$

and the breaking of the nilpotency of  $s_0$  is immediately seen by inserting the BRST equations, in the form (2.11), into the Bianchi identities (2.10) and isolating the

terms with ghost number 2. One obtains [7, 14]

$$s_0^2 B_2 = -[F, B_0^2]. \quad (2.12)$$

Thus, as announced earlier, the BRST operator is nilpotent only modulo the field equation  $F=0$ . This forbids the construction of a BRST invariant gauge fixed action by addition of a gauge fixing term of the form  $s_0$  (something) to the classical action. In ref. [14] a modification of the gauge symmetry has been proposed to cure this problem. In this paper, however, we shall follow the method of Batalin and Vilkovisky.

In accordance with the Batalin-Vilkovisky formalism, we thus introduce anti-fields for all the fields collected in the large pyramid of (2.4) as follows:

$$\begin{aligned} B_2^0 &\rightarrow B_2^{-1}, & B_1^{-1} &\rightarrow B_3^0, \\ B_1^1 &\rightarrow B_3^{-2}, & B_0^{-2} &\rightarrow B_4^1, \\ B_0^2 &\rightarrow B_4^{-3}, & B_0^0 &\rightarrow B_4^{-1}. \end{aligned} \quad (2.13)$$

It is suggestive to display all fields and anti-fields in a diagram as follows:

$$\begin{array}{cccccc} & & B_4^{-1} & B_0^0 & & \\ & & B_4^1 & B_3^0 & B_1^{-1} & B_0^{-2} \\ B_4^{-3} & B_3^{-2} & B_2^{-1} & B_2^0 & B_1^1 & B_0^2 \end{array} \quad (2.14)$$

The sum of the ghost numbers of any field and its anti-field is  $-1$ , in accordance with the Batalin-Vilkovisky formalism. By duality we have chosen the form degree of the anti-field of a field with form degree  $l$  to be  $d-l$ , where  $d$  is the dimension of spacetime. Now that the total degree of each anti-field in the geometric sector is 1, it is natural to combine them with the gauge field  $A$ , which also has this property (since  $A$  can be rewritten as  $A_1^0$ ), to define the unified object:

$$\mathcal{A} \equiv A + B_2^{-1} + B_3^{-2} + B_4^{-3}. \quad (2.15)$$

The generalized curvature now reads

$$\begin{aligned} \hat{\mathcal{G}} &\equiv (d+s)\mathcal{B} + [\mathcal{A}, \mathcal{B}] = \mathcal{D}\mathcal{B}, \\ \hat{\mathcal{F}} &\equiv (d+s)\mathcal{A} + \frac{1}{2}[\mathcal{A}, \mathcal{A}], \end{aligned} \quad (2.16)$$

where the BRST operator  $s$  will now act as a differential operator on all fields and antifields (with the assumption  $sd + ds = 0$ ), and will be shortly defined such that it

is nilpotent without the use of any field equation. To see how this can be done by imposing constraints on the generalized curvatures, we consider the Bianchi identities they obey:

$$\begin{aligned}(d+s)\hat{\mathcal{G}} &= s^2\mathcal{B} + [\hat{\mathcal{F}}, \mathcal{B}], \\ (d+s)\hat{\mathcal{F}} &= s^2\mathcal{A} + [\hat{\mathcal{F}}, \mathcal{A}].\end{aligned}\quad (2.17)$$

We see that, in order to have  $s^2 = 0$  on all fields and anti-fields, it is necessary and sufficient to impose the constraint that all components in  $\hat{\mathcal{G}}$  with positive ghost number are zero, and that  $\mathcal{F}$  vanishes:

$$\hat{\mathcal{G}}_{3-g}^g = 0, \quad \text{for } g > 0, \quad \mathcal{F} = 0. \quad (2.18)$$

These equations are the “off-shell” generalization of (2.8). By their expansion in ghost number one obtains the following nilpotent BRST transformations

$$\begin{aligned}s\mathcal{A} &= 0, \\ sB_2^0 &= -DB_1^1 - [B_2^{-1}, B_0^2], & sB_2^{-1} &= F, \\ sB_1^1 &= -DB_0^2, & sB_3^{-2} &= -DB_2^1, \\ sB_0^2 &= 0, & sB_4^{-3} &= -DB_3^2 - \tfrac{1}{2}[B_2^{-1}, B_2^{-1}].\end{aligned}\quad (2.19)$$

The BRST transformation rules for the nongeometrical fields, including the Lagrange multipliers, are as before (see eq. (2.6b)). The ghost number zero part of  $\hat{\mathcal{G}}$  defines the physical field strength  $\hat{G}$  which contains the anti-fields as follows

$$\hat{G} = G + [B_1^1, B_2^{-1}] + [B_0^2, B_3^{-2}]. \quad (2.20)$$

The ghost number zero part of the first Bianchi identity in (2.15) gives the interesting result that  $\hat{G}$  is BRST invariant:

$$s\hat{G} = 0. \quad (2.21)$$

We now turn to the construction of a BRST invariant action  $S$  which depends on the fields and anti-fields. Since we have constructed a nilpotent BRST operator  $s$  acting on the fields  $\phi$  and  $\phi^*$  the theorem of Batalin-Vilkovisky [1] guarantees the existence of an  $s$ -invariant action  $S(\phi, \phi^*)$  with

$$s\phi = \frac{\delta S[\phi, \phi^*]}{\delta \phi^*}, \quad s\phi^* = \frac{\delta S[\phi, \phi^*]}{\delta \phi}, \quad sS[\phi, \phi^*] = 0. \quad (2.22)$$

An  $s$ -invariant action is then

$$S = I_{\text{geometrical}} + \int \text{Tr} [B_3^0 \wedge \Pi_1^0 + B_4^{-1} \wedge \Pi_0^1 + B_4^1 \wedge \Pi_0^{-1}], \quad (2.23)$$

where  $I_{\text{geometrical}}$  is the part of the action which depends only on the geometrical sector and is given by

$$I_{\text{geometrical}} = I_{\text{cl}}(A, B_2^0) + \int \text{Tr} [B_2^{-1} \wedge (DB_1^1 + \tfrac{1}{2}[B_0^2, B_2^{-1}]) + B_3^{-2} \wedge DB_0^2]. \quad (2.24)$$

In order to obtain eventually Feynman type gauges, one may add to  $S$  the following trivially  $s$ -invariant term

$$I_F = \int d^4x (\tfrac{1}{2}\alpha \Pi_\mu \Pi^\mu + \beta \Pi_0^{-1} \Pi_0^1), \quad (2.25)$$

where  $\alpha$  and  $\beta$  are arbitrarily chosen gauge parameters.

The next step in the Batalin-Vilkovisky formalism is the elimination of the antifields by introducing a gauge function  $\psi^{-1}(\phi)$  of degree 0 and ghost number  $-1$ , and imposing the constraint  $\phi^* = \delta\psi^{-1}/\delta\phi$ . In order to obtain covariant gauges for the 2-form gauge field and its 1-form ghosts we choose

$$\psi^{-1} = B_\nu^{-1} (\partial_\mu B^{\mu\nu} + \partial^\nu B_0^0) + B_0^{-2} (\partial^\mu B_\mu^1). \quad (2.26)$$

Notice that the construction of  $\psi^{-1}$  necessitates the existence of the antighosts, i.e. the introduction of the nongeometrical sector. With the above choice of the gauge function the anti-fields are thus constrained as follows

$$\begin{aligned} B_2^{-1} &= dB_1^{-1}, \\ B_{\mu\nu\rho}^{-2} &= \varepsilon_{\mu\nu\rho\sigma} \partial^\sigma B_0^{-2}, \\ B_{\mu\nu\rho\sigma}^1 &= \varepsilon_{\mu\nu\rho\sigma} \partial^\tau B_\tau^1, \\ B_{\mu\nu\rho\sigma}^{-1} &= \varepsilon_{\mu\nu\rho\sigma} \partial^\tau B_\tau^{-1}, \\ B_{\mu\nu\rho}^0 &= \varepsilon_{\mu\nu\rho\sigma} (\partial_\tau B^{\tau\sigma} + \partial^\sigma B_0^0), \\ B_4^{-3} &= 0. \end{aligned} \quad (2.27)$$

Using these equations we can eliminate the anti-fields in the action (2.23) and the



BRST transformation rules (2.19). This yields

$$\begin{aligned}
 S\left(\phi, \phi^* = \frac{\delta\psi^{-1}}{\delta\phi}\right) = & \text{Tr} \int d^4x \left[ \frac{1}{2} A_\mu A^\mu + \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} B_{\rho\sigma} + D_{[\mu} B_{\nu]}^1 \partial_{[\mu} B_{\nu]}^{-1} \right. \\
 & + \varepsilon^{\mu\nu\rho\sigma} B_0^2 \left[ \partial_\mu B_\nu^{-1}, \partial_\rho B_\sigma^{-1} \right] + \partial^\sigma B_0^{-2} D_\sigma B_0^2 \\
 & + \Pi_\nu \left( \partial_\mu B^{\mu\nu} + \partial^\nu B_0^0 \right) + \Pi_0^{-1} \partial^\mu B_\mu^1 \\
 & \left. + \Pi_0^1 \partial^\mu B_\mu^1 + \frac{1}{2} \alpha \Pi_\mu \Pi^\mu + \beta \Pi_0^{-1} \Pi_0^1 \right], \quad (2.28)
 \end{aligned}$$

$$\begin{aligned}
 \underline{s}B_2 &= -DB_1^1 - [dB_1^{-1}, B_0^2], & \underline{s}A &= 0, \\
 \underline{s}B_1^1 &= -DB_0^2, \\
 \underline{s}B_0^2 &= 0, \\
 \underline{s}B_1^{-1} &= \Pi_1^0, & \underline{s}\Pi_1^0 &= 0, \\
 \underline{s}B_0^{-2} &= \Pi_0^{-1}, & \underline{s}\Pi_0^{-1} &= 0, \\
 \underline{s}B_0^0 &= \Pi_0^1, & \underline{s}\Pi_0^1 &= 0, \quad (2.29)
 \end{aligned}$$

with  $\underline{s}S(\phi, \phi^* = \delta\psi^{-1}/\delta\phi) = 0$ . The action (2.28) is gauge fixed (i.e. all propagators are defined) and it is invariant under the BRST transformations (2.29). This action is characterized by the presence of cubic ghost couplings. It is the same as that derived by De Alwis, Grisaru and Mezincescu by using a Noether inspired method [2] and has also been given in [15]. The authors of [2] have checked the formal unitarity theorem of Batalin and Vilkovisky by investigating the unitarity of the physical sector of the perturbative theory generated by the action (2.28), through a generalization of 't Hooft and Veltman's diagrammatical method [16].

As an application of the geometric structure of the BRST transformations which we have introduced, we can determine consistent anomalies, i.e. we can find solutions of the Wess-Zumino consistency equation [7, 8]:

$$s\Omega_4^1(B^g, B^{-1-g}) + d\Omega_3^2 = 0. \quad (2.30)$$

One solution to this equation follows from the existence of the following object

$$\Omega_6 = \text{Tr}(\mathcal{A}\mathcal{A}\mathcal{A}\mathcal{G}), \quad (2.31)$$

which is algebraically equal to

$$\Omega_6 = (d + s) \text{Tr}(\mathcal{A}\mathcal{A}\mathcal{A}\mathcal{B}), \quad (2.32)$$

provided that the Yang-Mills gauge group has no third rank invariant tensor with two indices that are symmetric. It is indeed obvious from (2.15) and (2.18) that  $\Omega_6$  has a vanishing component with ghost number 2 so that the component with ghost number 2 in (2.32) determines  $\Omega_4^1$  and  $\Omega_3^2$  as follows

$$\begin{aligned} \Omega_4^1 &= \text{Tr}(AAAB_1^1 + 3B_2^{-1}AAB_0^2), \\ \Omega_3^2 &= \text{Tr}(AAAB_0^2). \end{aligned} \quad (2.33)$$

Since  $sB_{\mu\nu} = \partial_{[\mu}B_{\nu]} + \dots$ , the potentially anomalous diagram corresponding to  $\Omega_4^1$  is simply  $\langle T(A_{[\mu}A_{\nu]}A_{\rho} \partial^\tau B_{\sigma]\tau}) \rangle_{1\text{PI}}$ . For anomaly freedom it is necessary that the  $\varepsilon_{\mu\nu\rho\sigma}$  structure in this Green function vanishes. Note that even in the absence of chiral fermions such a structure can be generated through the Feynman rules, since the action contains the parity odd term  $B \wedge F$ . After the elimination of the anti-fields,  $\Omega_4^1 = \text{Tr}(AAAB_1^1)$  remains the solution of the consistency equation for the operator  $\underline{s}$  modulo the equation of motion  $F = 0$ .

Finally we remark that the BRST quantization of the Freedman-Townsend model with a charged 2-form in 4-dimensional spacetime which we have performed above can be easily extended to the case of similar models with a charged  $p$ -form in  $(p+2)$ -dimensional spacetime. It is also possible to consider the case when the 1-form  $A$  is a gauge field. The starting point is then the action  $\int \text{Tr}(F_{\mu\nu}F^{\mu\nu}d^{p+2}x + B_p \wedge F)$  with the gauge invariance  $\delta A_\mu = D_\mu \varepsilon$ ,  $\delta B_p = D\varepsilon_{p-1}$ . The spectrum of fields and anti-fields for a  $p$ -form gauge field is displayed in fig. 1a.

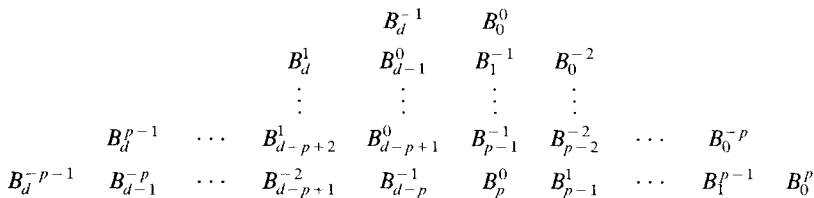


Fig. 1a. The field and anti-field spectrum for the  $p$ -form gauge field. A vertical line between  $B^{-1}$  and  $B^0$  would be a symmetry axis for the fields on the right hand side and the anti-fields on the left hand side. The base line represents the geometrical sector. The upper half plane contains all the antighost fields and their anti-fields. The figure is for even  $p$ . For odd  $p$  the top line would be  $(B_0^0 \ B_d^{-1})$  instead of  $(B_d^{-1} \ B_0^0)$ . The objects with positive ghost number  $g$  are fields on even layers and anti-fields on odd layers.

### 3. String field theory

#### 3.1. THE GEOMETRICAL SECTOR AND THE ALGEBRAIC STRUCTURE OF THE NILPOTENT BRST SYMMETRY

Open string field theory has a quasi-Yang-Mills structure in the free as well as the interacting case [10, 12]. Moreover, it bears some resemblance with the theory of charged forms because it exhibits the phenomenon of ghost for ghost and also because in the interacting case the gauge transformations admit zero modes modulo a field equation which is a vanishing field strength [17]. The problem of building a nilpotent BRST operator in the interacting case has been addressed in ref. [17] where some auxiliary fields have been introduced, and also in [3, 4] where Bochichio and Thorn have applied the Batalin-Vilkovisky method. We will study again this problem in the light of a geometric analysis similar to that we have used for the charged 2-form. We will see in particular that the usual interpretation of the antighost sector of the theory has to be revised.

In order to establish a covariant formalism of the free string field theory, it is known [10, 11] that one must introduce all string fields occurring in the expansion of a functional  $\chi[X^\mu(\sigma), c(\sigma), \bar{c}(\sigma)]$  where  $X^\mu$  denotes the string coordinate, and  $c, \bar{c}$  are a pair of ghost coordinates. The expansion of  $\chi$  in ghost number determines a set of string fields  $A^g[X^\mu]$ , where the ghost number  $g$  varies between  $-\infty$  and  $+\infty$ .  $A^0$  is the classical string field,  $A^1$  is the primary ghost,  $A^2$  is the ghost for ghost, etc. The ghosts  $A^g$  with negative  $g$  have been interpreted as antighosts, while the Lagrange multiplier string field  $\Pi^{g+1}(X^\mu)$  has been associated with each “antighost”  $A^g$  ( $g < 0$ ). The following set of fields were therefore considered as the fundamental fields of the string field theory:

$$\begin{array}{cccccccccccc} \dots & A^{-g} & \dots & A^{-3} & A^{-2} & A^{-1} & A^0 & A^1 & A^2 & \dots & A^g & \dots \\ \dots & \Pi^{-g+1} & \dots & \Pi^{-2} & \Pi^{-1} & \Pi^0 & & & & & & \end{array} \quad (3.1)$$

The classical action for open string field theory is [11, 12]

$$I_{\text{cl}} = \int A^0 * Q A^0 + \frac{2}{3} A^0 * A^0 * A^0, \quad (3.2)$$

where  $Q$  is a nilpotent operator representing the Virasoro algebra in 26 dimensional spacetime. The associative graded product  $*$  and the integration symbol  $\int$  have been constructed in [11]. All fields  $A^g$  have odd grading, say 1. The auxiliary fields  $\Pi^g$  have grading 2, and  $Q$  has grading 1. The basic properties are:  $Q(\text{anything}) = 0$ , and  $Q(X * Y) = (QX) * Y \pm X * (QY)$ , where the minus sign occurs when  $Y$  has odd grading.

$I_{\text{cl}}[A]$  is formally identical to a Chern-Simons form and is invariant under the following gauge transformation

$$\delta A^0 = Q\epsilon + [A^0, \epsilon], \quad (3.3)$$

where we have used the notation

$$[A, B] = A * B + (-)^{ab} B * A, \quad (3.4)$$

where  $a, b$  are the gradings of  $A, B$ , respectively. From (3.3) we see that  $\varepsilon$  of the form  $D\varepsilon'$ , for any  $\varepsilon'$ , is a zero mode of the gauge transformation, modulo the field equation  $F = QA^0 + A^0 * A^0 = 0$ . But  $\varepsilon'$  itself admits a zero mode of the form  $D\varepsilon''$  for any  $\varepsilon''$ , modulo the field equation  $F = 0$ , and so on. Hence we see the similarity between the string field theory and that of the charged forms which we treated in the previous section.

A further similarity of the string field theory with the theory of charged forms is the breakdown of the nilpotency of the BRST transformations when the string fields  $A^g (g \geq 0)$  are considered as building the geometric fields. In this case the BRST symmetry for the geometrical sector is defined as follows [17]

$$\begin{aligned} s_0 A^0 &= -QA^0 - [A^0, A^1], \\ s_0 A^1 &= -QA^1 - [A^0, A^2] - \tfrac{1}{2}[A^1, A^1], \\ &\vdots \\ s_0 A^g &= -QA^{g+1} - \sum_{n=0}^{g+1} A^n * A^{g+1-n}, \\ &\vdots \end{aligned} \quad (3.5)$$

while for the antighost sector one has

$$s_0 A^g = \Pi^{g+1}, \quad s_0 \Pi^{g+1} = 0, \quad g < 0. \quad (3.6)$$

In the geometrical sector it is natural to define [17]

$$\begin{aligned} \mathcal{A} &= \sum_{g=0}^{\infty} A^g, \\ \mathcal{F} &= (Q + s_0)\mathcal{A} + \mathcal{A} * \mathcal{A}. \end{aligned} \quad (3.7)$$

Then, the geometrical BRST equations (3.5) can be rewritten as

$$\mathcal{F} = F. \quad (3.8)$$

Assuming that  $Q$  and  $s_0$  anticommute, one has  $(Q + s_0)^2 = s_0^2$  and the Bianchi

identity which is the consequence of the associativity of the star product gives

$$(Q + s_0) \mathcal{F} = s_0^2 \mathcal{A} - [\mathcal{A}, \mathcal{F}], \quad (3.9a)$$

from which, by using (3.8), it follows that

$$(Q + s_0) F = s_0^2 \mathcal{A} - [\mathcal{A}, \mathcal{F}]. \quad (3.9b)$$

By isolating in (3.9b) the terms with ghost number  $\geq 2$  one obtains [17]

$$s_0^2 A^g = [A^{g+2}, F], \quad (3.10)$$

which means that the operator  $s_0$  is only nilpotent on-shell, i.e. for  $F = 0$ .

In order to cure this problem one can introduce anti-fields. But such anti-fields are already present in the theory if one abandons the interpretation of the fields  $A^g$  for  $g < 0$  as antighosts. We shall therefore change the geometrical unification (3.7) into the following one

$$\hat{\mathcal{A}} = \sum_{g=-\infty}^{+\infty} A^g. \quad (3.11)$$

In this equation  $A^{-g-1}$  stands for the anti-fields of  $A^g$  for any  $g \geq 0$ . The price one has to pay for this identification is that new objects will be needed for playing the role of the antighosts. The antighosts are needed in particular for writing a gauge fixed BRST invariant action in a general gauge. The field strength of the unified object  $\hat{\mathcal{A}}$  is

$$\hat{\mathcal{F}} = (Q + s) \hat{\mathcal{A}} + \frac{1}{2} [\hat{\mathcal{A}}, \hat{\mathcal{A}}], \quad (3.12)$$

where  $s$  is now expected to be an operator which is nilpotent independently of any field equation, thanks to the presence of the anti-fields in  $\hat{\mathcal{F}}$ . The Bianchi identity reads

$$(Q + s) \hat{\mathcal{F}} = s^2 \hat{\mathcal{A}} + [\hat{\mathcal{A}}, \hat{\mathcal{F}}]. \quad (3.13)$$

In order to have  $s^2 = 0$  on all components of  $\hat{\mathcal{A}}$ , it is clear from the last equation that the definition of  $s$  must be

$$\hat{\mathcal{F}} = 0. \quad (3.14)$$

By substituting (3.14) into (3.12), the transformation rules can be read by a straightforward expansion in ghost number. For the classical string field one has

$$sA^0 = -QA^1 - [A^1, A^0] - \frac{1}{2} \sum_{g>0} [A^{-g+1}, A^g]. \quad (3.15)$$

If one sets in this equation all the anti-fields equal to zero one recovers the definition (3.8) of the “on-shell” BRST operator  $s_0$ . Moreover, one has the following generic formula for the action of  $s$  on all fields and anti-fields  $A^g$ , equivalent to (3.14):

$$sA^g = -QA^{g+1} - \sum_{n=-\infty}^{+\infty} A^n * A^{g+1-n}. \quad (3.16)$$

One can verify that  $s^2 = 0$  directly from this equation. In fact, this is guaranteed by construction.

### 3.2. THE FIELD AND ANTI-FIELD DEPENDENT INVARIANT ACTION

Since we have built the operator  $s$  with  $s^2 = 0$ , an  $s$ -invariant action  $S[A^g]$  must exist, the equations of motions of which yield the definition (3.14) of the BRST symmetry. The following action

$$I[\hat{\mathcal{A}}] = -\frac{1}{2} \int (\hat{\mathcal{A}} * Q\hat{\mathcal{A}} + \frac{2}{3} \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}})_{g=0}, \quad (3.17)$$

is such that for any value of  $g$

$$\frac{\delta I}{\delta A^{-g-1}} = s\hat{\mathcal{A}}^g = - (Q\hat{\mathcal{A}} + \hat{\mathcal{A}} * \hat{\mathcal{A}})^{g+1}. \quad (3.18)$$

Eq. (3.18) is equivalent to the definition of  $s$  in (3.14). Now we employ the general theorem of Batalin and Vilkovisky [1] which states that if an action  $S(\phi, \phi^*)$  satisfies

$$s\phi = \frac{\delta S(\phi, \phi^*)}{\delta \phi^*},$$

$$s\phi^* = \frac{\delta S(\phi, \phi^*)}{\delta \phi},$$

$$s^2 = 0, \quad (3.19)$$

then,  $sS(\phi, \phi^*) = 0$ . Therefore the action (3.17) is invariant under the BRST transformations (3.14). Another more direct construction of the  $s$ -invariant action (3.17) is the following one. One has the identity

$$\begin{aligned} 0 &= \int \hat{\mathcal{F}} * \hat{\mathcal{F}} = \int (Q + s) * [\hat{\mathcal{A}}(Q + s) \hat{\mathcal{A}} + \frac{2}{3} \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}}] \\ &= s \int [\hat{\mathcal{A}} * (Q + s) \hat{\mathcal{A}} + \frac{2}{3} \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}}]. \end{aligned} \quad (3.20)$$

Besides, we already know that  $s^2 = 0$  and thus:

$$\begin{aligned}
 s \int \hat{\mathcal{A}} * s\hat{\mathcal{A}} &= \int s\hat{\mathcal{A}} * s\hat{\mathcal{A}} \\
 &= \int (Q\hat{\mathcal{A}} + \hat{\mathcal{A}} * \hat{\mathcal{A}}) * (Q\hat{\mathcal{A}} + \hat{\mathcal{A}} * \hat{\mathcal{A}}) \\
 &= \int Q \left[ \hat{\mathcal{A}} * Q\hat{\mathcal{A}} + \frac{2}{3} \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}} \right] = 0.
 \end{aligned} \tag{3.21}$$

One has therefore identically

$$s \int \left[ \hat{\mathcal{A}} * Q\hat{\mathcal{A}} + \frac{2}{3} \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}} \right] = 0. \tag{3.22}$$

This equation proves that not only the part with ghost number zero in  $\int [\hat{\mathcal{A}} * Q\hat{\mathcal{A}} + \frac{2}{3} \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}}]$  is  $s$ -invariant, but also that all components of any given ghost number are  $s$ -invariant. The existence of the  $s$ -invariant term with ghost number 1,  $\int [\hat{\mathcal{A}} * Q\hat{\mathcal{A}} + \frac{2}{3} \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}}]_{g=1} \approx -\frac{1}{3} \int \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}}_{g=1}$  modulo  $s$ -invariant terms, might be a signal of a potential anomaly in string field theory. The existence of anomalies in string field theory has been suggested in [18].

Before the elimination of the anti-fields, it is amusing to observe that both terms in (3.23) are separately  $s$ -invariant. This can be easily seen by writing

$$\begin{aligned}
 I[\hat{\mathcal{A}}] &= -\frac{1}{2} \int \left( \hat{\mathcal{A}} * Q\hat{\mathcal{A}} + \frac{2}{3} \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}} \right)_{g=0} \\
 &= -\frac{1}{2} \int \left( \hat{\mathcal{A}} * \hat{\mathcal{F}} - \frac{1}{3} \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}} - \hat{\mathcal{A}} * s\hat{\mathcal{A}} \right)_{g=0} \\
 &= \frac{1}{6} \int \left( \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}} + 3\hat{\mathcal{A}} * s\hat{\mathcal{A}} \right)_{g=0}
 \end{aligned} \tag{3.23}$$

where we have applied the definition of the BRST symmetry  $\hat{\mathcal{F}} = 0$ . It is indeed obvious from (3.23) and (3.21) that since  $I$  is  $s$ -invariant,  $\int (\hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}})_{g=0}$  is  $s$ -invariant by itself, and thus also  $\int (\hat{\mathcal{A}} * Q\hat{\mathcal{A}})_{g=0}$ .

### 3.3. THE ANTIGHOSTS AND THE ELIMINATION OF THE ANTI-FIELDS

So far we have built a nilpotent BRST operator which acts on the fields and anti-fields. Furthermore, we have constructed an invariant action depending on these fields. In order to eliminate the antifields, and arrive at a gauge fixed BRST invariant action, a gauge function  $\psi^{-1}$  must be introduced which has ghost number

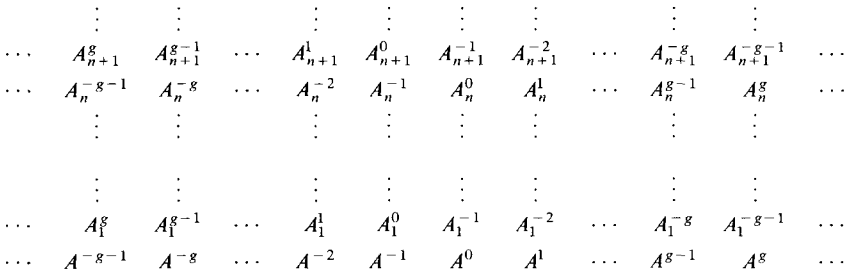


Fig. 1b. The ghost spectrum for the string field. The base line stands for the geometrical sector. The upper half plane contains the antighost string fields and their anti-fields. Here,  $n$  is an even integer. This figure is formally an infinite extension of fig. 1a.

$-1$ , and which does not depend on the anti-fields. This necessitates the introduction of anti-ghosts, i.e. fields with negative ghost number, together with their anti-fields. This means that we must fill the “upper half plane” on top of the base line which represents the geometrical objects  $A^g$  stemming from the ghost expansion of the unified string field  $\hat{\mathcal{A}}$ , as displayed in fig. 1b\*. The figure shows that the field spectrum of string field theory is a formal generalization, for  $p \rightarrow \infty$ , of the field spectrum which is displayed in fig. 1a and which is relevant for  $p$ -form gauge fields. Since  $\hat{\mathcal{A}}$  itself is obtained from a functional  $\chi(X^\mu, c, \bar{c})$ , we are thus led to introduce an infinite set of functionals  $\chi_n(X^\mu, c, \bar{c})$ , where  $1 \leq n \leq \infty$  stands for the antighost number. By expansion of each one of these functionals on a basis of ghost creation and annihilation operators one then obtains all the antighosts  $A_n^g$  ( $-\infty \leq g \leq \infty$ ,  $1 \leq n \leq \infty$ ) which fill the upper half plane as displayed in fig. 1b. This new set of string fields and anti-fields must of course be completed with Stueckelberg type fields  $\Pi_n^g$  with the following transformation rules

$$sA_n^g = \Pi_n^{g+1}, \quad s\Pi_n^{g+1} = 0. \quad (3.24)$$

As in the case of  $p$ -form gauge fields, the Stueckelberg fields  $\Pi_n^g$  are in one to one correspondence with the fields  $A_n^g$  of the antighost sector.

In earlier works [3,10,11] the existence of antighost string fields had not been pointed out. It is in fact possible to obtain the light-cone gauge and the Siegel gauge by starting from the covariant formulation of string field theory and considering only the field spectrum shown in (3.1). The possibility of not considering the full spectrum which includes what we call the true antighosts and their anti-fields, and nevertheless obtaining consistent actions in the above mentioned gauges, is comparable to the possibility of writing the Yang-Mills action in an axial gauge,

\* These antighosts had been introduced in [12] from other considerations based on the possibility of defining an anti-BRST operator in string field theory.



ignoring the existence of the Faddeev-Popov ghosts. In the Siegel gauge one builds a gauge function  $\psi^{-1}(\phi)$ , the negative ghost number of which is carried by the zero mode of the string ghost coordinate [3]. With the new antighosts  $A_n^g$  it becomes possible to construct gauge functions  $\psi^{-1}(A^g, A_n^g)$  which generalize the gauge function in (2.21) which we have considered in the case of 2-form gauge fields. We would call the corresponding gauges in string field theory “covariant gauges”. The corresponding gauge fixed BRST invariant action would be

$$I = \int \left[ \hat{\mathcal{A}} * Q \hat{\mathcal{A}} + \frac{2}{3} \hat{\mathcal{A}} * \hat{\mathcal{A}} * \hat{\mathcal{A}} + \sum_n A_n * \Pi_n \right]_{g=0}, \quad (3.25)$$

where all anti-fields  $\phi^* \equiv (A^g, A_n^g)_{g < 0}$  are now restricted to be  $\phi^* = \delta\psi^{-1}/\delta\phi$  with  $\phi \equiv (A^g, A_n^g)_{g \geq 0}$ .

#### 4. Conclusions

We have seen that the formalism of Batalin and Vilkovisky for gauge symmetries with open BRST algebra exhibits interesting geometric features. A question which we have left open is the consequences of the anomaly candidates which we have found prior to the elimination of the anti-fields, both in the Freedman-Townsend model and in string field theory. An output of our results concerning the application of the Batalin-Vilkovisky formalism in string field theory is the introduction of what we have called antighost string fields. Further use of these new objects could lead to the construction of new “covariant gauges” as substitutes to the light-cone gauge or the Siegel gauge.

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#### Note Added

Since the submission of this paper for publication, we have learnt the existence of refs. [19–21] which deal with the quantization of the antisymmetric tensor gauge theories.

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